HULL BENDING MOMENT DUE TO SPRINGING

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ABSTRACT

As a starting point for the research about the phenomenon of springing, a careful survey of the existing literature on this subject was made.

A mathematical model for the calculation of the bending moment due to springing is proposed. This mathematical model uses the modal analysis of the vibration of the ship and considers the effect of the distortion of the exciting wave along the hull for the calculation of the loading function.

The influence of some design parameters on the occurrence of springing is pointed out, and it is suggested some further research on the problem, with the aim of evaluating the accuracy of the method developed.

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NOMENCLATURE

A	- section area
$A_o \ldots A_n$	- numerical constants
A_{33}	- sectional vertical added mass
B	- local beam
c	- structural damping coefficient per unit length (function of x)
	- also wave celerity
C_i	- generalized damping
C_s	- local section area coefficient
c_s	- structural damping (due to mode frequency only)
E	- modulus of elasticity
EI	- bending flexural rigidity
G	- shear modulus
g	- acceleration of gravity
H	- local section draft
$\overline{h}$	- mean draft of the section
I	- sectional area moment of inertia
i	- mode of vibration
I_r	- mass moment of inertia per unit length with respect to an axis normal to the [illegible] plane
$J_r(\partial^2\gamma/\partial t^2)$	- rotary inertia
I_o	- Bessel function of 0 order
j	- imaginary unit
K	- ratio of the average shear stress to the shear stress at the neutral axis under vertical load

k - wave number

KAG - vertical shear rigidity

K_i - generalized spring constant

L - ship's length

M - bending moment

M_i - weighting function for bending moment

M_s - bending moment due to springing

$N_z(x)$ - local damping force coefficient

N_o - Newman function of 0 order

$P(x,t)$ - total force per unit length due to ship wave interaction

$Q_i(t)$ - weighting forcing function for the i^{th} mode

$q_i(t)$ - time-varying beam deflection for the i^{th} mode, in a generalized coordinate system

t - time

V - ship's forward speed

V_s - shear force

V_{si} - weighting function for shear force

$X_i(x)$ - normal node shape for the i^{th} mode

x - horizontal axis in direction of forward motion of the ship (along length of ship)

y - horizontal axis perpendicular to x

z - vertical elastic deflection

- also vertical axis

α - angle between the ship's heading and the wave direction, $\alpha=0$ for head seas

ϕ - wave potential

γ - elastic deformation angle

λ - wave length

λ_e - effective wave length

λ_i - parameter defined by equation (28)

μ - effective mass per unit length, which is the sum of the ship's mass and the added mass (high frequency limit)

- also coefficient defined by equation (41)

$\bar{\mu}_i$ - generalized mass

μ_1 - frictional coefficient

η - undistorted wave profile

η^* - effective wave profile

$\vec{\eta}$ - unit vector normal to the hull's surface

Ω_o - Lommel function of 0 order

ω - circular frequency

ω_e - frequency of encounter

ω_i - natural frequency of the i^{th} mode

ρ - specific gravity

ν - viscosity

$(\frac{D}{Dt})$ - substantial derivative

$(\frac{\partial}{\partial t})$ - partial derivative with respect to time

$(\frac{\partial}{\partial x})$ - partial derivative with respect to space

$(\dot{\ })$ - derivative with respect to time

$(\ )'$ - derivative with respect to space

[ ] - references in Chapter 4

( ) - equation's number

1. INTRODUCTION

1.1-WAVE INDUCED STRUCTURAL LOADS

In order to calculate the ship's hull structural response in a seaway, it is necessary to evaluate the loads both in magnitude and in phase, so as to make possible to superimpose them.

The above mentioned loads can be generally considered in two groups: the wave loads that vary slowly with time include the stillwater bending and shear, the wave bending moment and shear, and torsion; and the wave-induced vibratory loads, of a higher frequency, corresponding to the dynamic loading of the hull.

The stillwater bending moment and shear is readily calculated by available methods [1]. The evaluation of the wave bending moment and shear involves the calculation of the ship's response to regular and irregular seas, as well as both the short-term and long-term responses.

When a ship is moving in a seaway, five different types of forces contribute to the total loading: hydrodynamic, hydrostatic, gravity, inertia and viscous forces. It is usual to neglect the viscous forces since they are relatively small. The hydrostatic, gravity and inertia forces have been calculated to a good degree of accuracy for the last few years. The hydrodynamic loads are of a very complex nature, and their evaluation requires the calculation of the pressure distribution around the ship's hull in a seaway. This pressure

distribution is then used for determining vertical and lateral bending moments.

Theoretical techniques [2,3] are now available to evaluate motions and loads of conventional ships for five degrees of freedom (roll, pitch, yaw, heave and sway).

The low frequency wave-induced loads are considered in the papers mentioned above in both vertical and lateral directions, with the ship going in any heading relative to the wave direction. Lateral bending moment, vertical bending moment, torsional moment and shear force are calculated using the results of the motions of heave and pitch for the vertical bending moment and shear, and the motions of sway, yaw and roll for the lateral bending moment and torsional moment.

Torsional strength has become very important for the design of containerships because of their large hatch openings, therefore susceptible to structural damage due to torsion especially in quartering seaways. The analysis carried out in [2] show the distribution of torsional moment over the ship length. It is important to note that the maximum torsional moment doesn't necessarily occur amidships, but may occur at a considerable distance aft of it. There is also an important correlation between the rolling motion and torsional moments, making the accurate calculation of the roll motion a must.

In addition to the low frequency wave loads discussed above, higher frequency wave induced vibratory loads are often encountered by ships in a seaway. These vibratory loads are

caused by a mechanism totally different from the one that generates the low frequency wave loads, therefore resulting in a completely independent response whose forces may, under certain conditions, increase the maximum vertical bending moment to considerably high values.

This vibratory response is sometimes due to large ship motions resulting in the emmersion of the bow or stern regions, which leads to an impact force when the hull enters the water (slamming).

In ships with large bow flare, the forces resulting from the bow flare shape variations and the motions of the ship's hull often lead to a vibration known as "whipping".

We have therefore a system of forces dependent on hydrodynamic non-linear effects and the motions of a ship's hull considered rigid, with a response that can be approximated as linear. The resulting response depends also on the different frequencies and shapes of the vibratory behavior of the hull (structural modes), and appears as a non-stationary record of high frequency oscillations resulting from the impulsive local forces at the points of impact between the hull and the water surface. The frequency of this vibratory motion is usually that of the first structural mode (two-node vibration), because the resonance at higher natural modes is reduced due to structural and hydrodynamic damping.

It is necessary to consider now the way the wave bending and slamming loads are superimposed. Slamming stress records show an initial effect of a large compressive stress on the

deck amidships, in sagging. Usually this is not very important because it occurs when the wave bending moment changes from hogging to sagging and is therefore very small. The "whipping" vibratory stress continues after the slam, and a critical situation may occur when it is superimposed on the bottom plate with the wave bending moment peak in sagging. The critical condition for the deck occurs a half-cycle later, when the maximum hogging bending moment is superimposed to the remaining whipping stress. It is evident that these conditions become more dangerous with large stillwater bending moments.

Besides the evaluation of the highest stress due to slamming and whipping, it is also important to calculate the frequency of occurrence of slamming and the corresponding number of stress reversals expected, so that considerations on the fatigue performance of the hull can be made.

Another vibratory phenomenon denoted as springing has a somewhat different origin from slamming. It results from small motions of a ship in relatively short waves, when the frequency of encounter with the wave is comparable to the lower structural modes of vibration of the ship. Springing is specially important in fast ships such as destroyers and containerships, and in large tankers and bulk carriers. It behaves like a stochastic process similar to wave bending except that it occurs at higher frequencies. Springing can be analysed as a linear process, since it results from a resonant condition between the two-node vibratory bending response of the ship structure and the existing wave.

This phenomenon will be discussed in greater detail in the next section.

1.2-SPRINGING PHENOMENON

The occurrence of springing of a ship is considered to be due to the selective resonance of the ship hull vibration with the frequency of encounter of the wave. It is originated from the synchronism between the hull's natural frequency of vibration and a region of the wave energy spectrum of sufficiently high energy content to excite a response.

As said before, the particular ships for which springing assumes a greater importance are large tankers and bulk carriers, as well as fast ships such as destroyers and containerships.

The evaluation of the wave induced vibrations requires the calculation of the forces due to the exciting high-frequency waves, distributed along the length of the ship, and the elastic properties of the ship's hull including structural damping.

The vertical hull force per unit length is composed of three parts: hydrostatic, damping and inertia forces. If the damping assumes negative values, then the vibration will be self-exciting. Calculations carried out for several tankers show that the damping could be negative, resulting in a ship form more sensitive to springing. An extended accumulation of data from the springing effect in various ships could eventually show a possible correlation between

damping values and springing response. If this negative damping situation ever occurs, most likely the self-exciting situation will be only of a transient nature and unstable.

It has been found in previous treatments of the problem of springing that the damping (including the structural damping and the damping resultant from the distribution of added mass along the ship length) is relatively small, which leads to a very fine tuning in the response characteristics of the phenomenon.

Analysed data [4] seems to show that the springing stresses manifest themselves as a rather narrow band Gaussian phenomenon. Using the superposition principle the ship vibrations in irregular waves can be calculated, if the frequency characteristics and the energy spectrum of the waves are known. Since the springing stresses in regular waves appear to be linear with the amplitude of the waves, the response in irregular waves can therefore be evaluated by superposition of the stresses resulting from the regular wave components that constitute the irregular seaway, as indicated in [5].

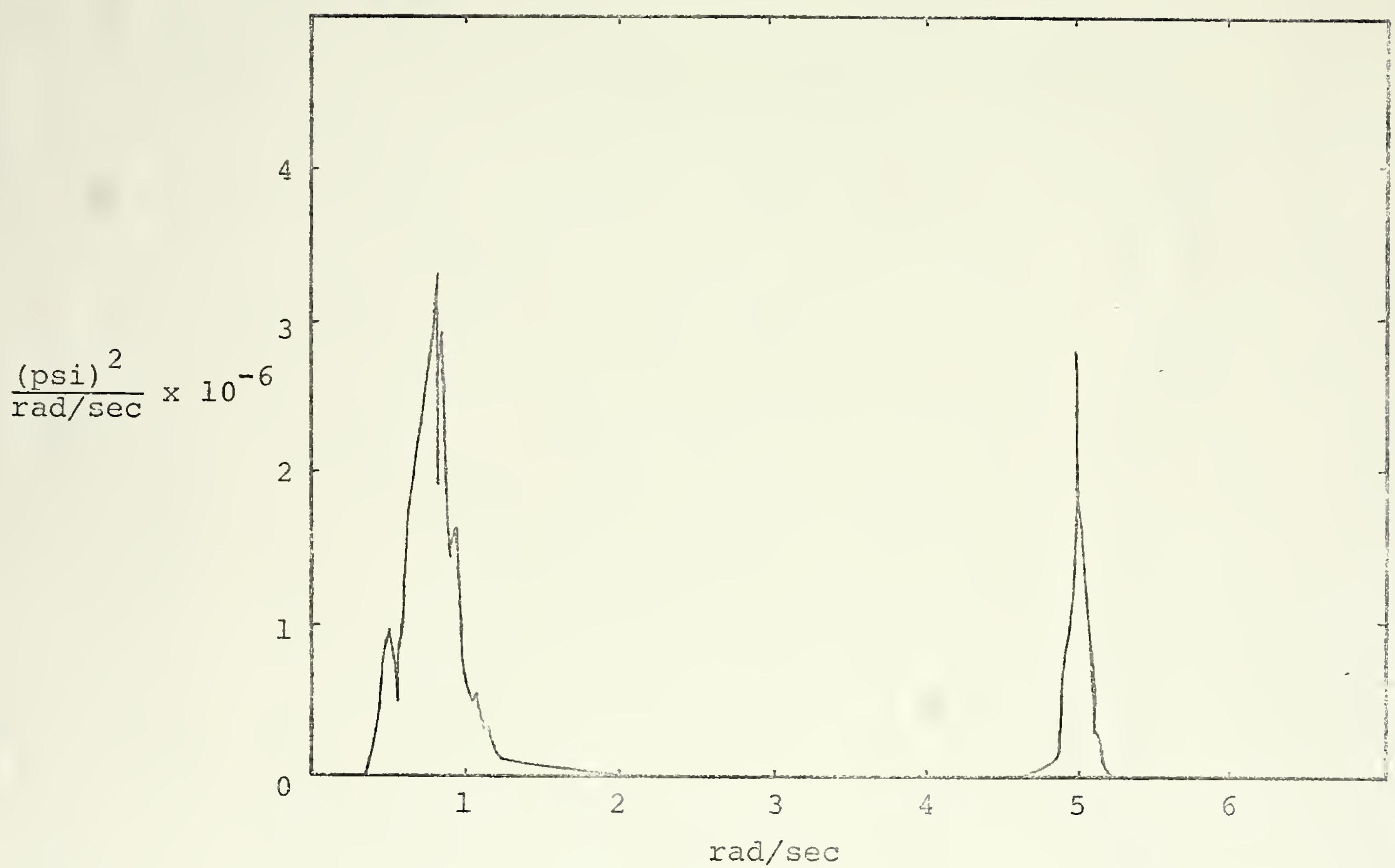
The presence of strong springing has been observed even in a low sea state. When the root mean square values of the springing bending moment are examined for a particular sea state it is apparent its great sensitivity to the ship's forward speed and heading. The principal reason for this sensitivity is that when the ship changes heading or speed it encounters a large range of wavelength values that can excite

the two node natural vibration of the hull. This sensitivity is also caused by the oscillatory variation of the amplitude of the exciting force, when the wavelength is changed.

Since the importance of springing was recognized, several reports of full scale high frequency stresses encountered on ships at sea have been published, in the hope of clarifying the nature of the phenomenon, as well as provide experimental grounds to check the various mathematical models proposed.

Stress data from measurements carried out on the Great Lakes ore carrier E.L. Ryerson show that springing stress variations could be as high as 64 percent of the wave bending stress variations and magnitudes of the springing stress on the order of 14,000 to 15,000 psi were found [6]. A typical stress-response spectrum indicating combined effects of wave induced and springing stresses is given in Figure 1. Tests on the same ship also revealed that a small change in heading or speed would change considerably the magnitude of the springing stress, eliminating it completely in many cases.

Usually springing stresses are observed much more often than long period wave stresses. Measurements taken on the Ryerson showed stresses of all magnitudes, including stresses nearly twice as large as the maximum low frequency wave stress. As the springing phenomenon has a high frequency of occurrence, it is most important to realize how it can affect the normal lifetime of a ship if the springing stress is high enough to affect the fatigue properties of the structure.



STRESS RESPONSE SPECTRUM FROM "ONTARIO POWER" [4]

FIGURE 1

The springing stress is an important third major stress to add to the normal still water and low frequency wave bending which have been the loads considered in traditional design methods. It was observed that this phenomenon was the source of the highest dynamic stresses in a normal shipping season on Great Lakes vessels.

1.3-LITERATURE SURVEY

Since the importance of the high frequency wave excited vibratory loading of the ship's hull was realized, a relatively large number of works have been published.

Earlier papers on the subject include those of BELGOVA [7] in 1962 and MAXIMADJI [8] in 1967, which deal with the vibratory response to regular waves.

MATHEWS [9], in 1967 extended his research to the vibratory response in irregular waves, with the analysis of information obtained on the "Ontario Power". He suggested that springing stresses were a function of the geometry of the hull, hull stiffness, speed and sea state, and noted the significant changes in springing response when the ship's heading was altered. His paper deals with the effect of wave excited vibration on ship longitudinal strength, in particular on the Great Lakes bulk carriers.

GOODMAN [10], in 1970, developed a theory for the prediction of springing based on the assumption that the phenomenon is the result of the two-noded hull vibration excited by component waves of the spectrum that correspond in frequency

to the natural frequency of the two-noded hull girder vibration. He assumes that there is always energy available in the sea spectrum to excite the vibration at the correct frequency. Since the period of oscillation increases with the length of the ship and the energy level available in the sea spectrum for vibration will in general increase for a fully developed sea, the presence of significant springing in very large ships in fully developed seas should be expected.

VAN GUNSTEREN [11] in 1970, derived a theory of wave induced main hull vibrations in regular and irregular waves. The vibrating hull is represented by a beam-element model, the wave exciting forces being determined with a strip theory for oblique waves. The beam model utilized has a discrete mass and stiffness distribution. The approach is somewhat similar to Goodman's, but here the bending moment is explicitly represented in the modal form, and also the response at frequencies lower than the two-node natural frequency are introduced.

KUMAI and TASAI [12], in 1970, calculate the hydrodynamic force induced by waves and which results in a high frequency hull vibration. The calculation is based on the measured hydrodynamic pressure at the bow of a 76,000 d.w.t. tanker, as a function of time.

ROBERTSON, CLEARLY and YAGLE [6] published, in 1971, a report with the results of strength studies carried over a period of four years on the Great Lakes bulk ore carrier "Edward L. Ryerson". The purpose of this investigation was

to obtain data of full scale bending response and the exciting waves, in order to allow the study of its correlation. Along with other important conclusions, the authors found that the vertical two-node longitudinal vibration (springing) was the most frequent dynamic loading of the hull girder. Dynamic stressing due to springing was found in 30 to 45 percent of the observations, and its magnitude was sufficiently high to deserve being included in the loading considerations for design.

MILES [4], presented in 1971 a study on the short-term statistical distribution of peaks which occur when low frequency wave induced stresses and higher frequency springing stresses are present simultaneously in a ship. The observations took place in periods from approximately twenty minutes to at most a few hours, so that the sea state could be considered a stationary random process. Springing was found to be a narrow band Gaussian phenomenon, with stress peaks highly correlated. Rice analysis of random noise [13] was successfully applied to the springing phenomenon.

LEWIS and WHEATON [14] (1971) utilized the work of Miles in the study of the combined wave and springing stresses of the "Edward L. Ryerson", looking for a rational manner of combining the still water bending, the low frequency wave induced bending and springing in order to arrive at reasonable strength requirements. It was found that pure wave bending stresses are always increased by springing, and that the range of maximum springing stresses is not affected greatly by the range of wave bending stress. The highest wave bending stress

and the highest springing stress in a twenty to thirty minute record seldom coincide in time and therefore the highest combined stress in any record is always less than the sum of the highest wave bending stress and the highest springing stress in the record. The authors of reference [14] point out the feasibility of predicting the maximum combined stress when the separate wave and springing stresses are known or calculated, and from there a long-term prediction of the combined stress, as previously done for wave bending only.

KAPLAN and SARGENT [15] (1972) evaluated the springing response assuming that the bending moment (due to springing) is represented in terms of the hull acceleration associated with the mode shape, hence eliminating significant contributions at lower frequencies. The results allow the presentation of a time history of the vertical bending moment for a particular exciting wave system time history, as well as a representation in spectral forms via frequency response methods. The root mean square values obtained using the two methods are in agreement for the same wave system, and therefore the choice of any one of the approaches depends upon the particular use of the results (for example, the spectral form will be used for long term prediction, and the time history for comparison with details of experiments involving precise phase relations). The solution of the hydrodynamic problem of high frequency local wave loading is attempted using a two-dimensional section approach, but in spite of its different mathematical form, the numerical results obtained show very

little difference from the treatment of present strip theory procedures for the case of a semi-circular cross section.

KUMAI published in 1972 a paper [16] on the ship's hull vibratory response to high frequency wave-induced loading, followed by another work [17] in 1973 in which the effect of distortion of the wave on the ship side is included. In the later work the wave induced vibratory force and response are evaluated using the ship side wave height as measured by self-propelled model test of a tanker in regular waves. The results seem to show that the exciting force is distributed along the ship length, although concentrated on the region of entrance of the hull because of the effect of the distorted wave, which in this case was two to three times higher at the bow and a half to a quarter lower at the stern than the undistorted wave. The author also derives an expression that shows that a ship with large beam to depth ratio will be subjected to more severe springing stresses.

KAPLAN et al [18] and NORDENSTROM et al [19] published in 1973 the reports of the Ship Structure Committee (Committees 2 and 3 respectively), where recent developments on the evaluation of wave loads are discussed to a great extent. The Report of Committee 2 concentrates on the hydrodynamic aspects of the wave loading (both low and high frequency), while the Report of Committee 3 offers a statistical approach to the problem. Both reports offer extended lists of references.

Tests on springing are being carried out at the Webb Institute of Naval Architecture in order to continue the investigation using models and allow further computations in an effort to obtain some correlation between theory and experiment. The Shipbuilding Laboratory at Delft is planning tests with a segmented model of a tanker in order to obtain data on springing. Hopefully the results from these and other tests will help the understanding of the fundamental parameters that affect the high frequency vibratory response of ships and also the sensitivity of the results to the assumptions used in current mathematical models.

2. MATHEMATICAL MODEL

2.1-GENERAL

We will proceed now to find a system of equations that will provide an adequate description of the ship structural response to springing. The two following steps would be the conversion of these equations into a computer program and the evaluation of the proposed mathematical model by comparing its predictions with experimental results; these two steps arebeyond the scope of the present work and therefore will not be carried out here.

When considering the problem of the vibration of a ship hull, it is usual to consider the hull structure as an elastic beam with free ends and nonuniform mass along the ship's length. The vibration is therefore governed by the Timoshenko equation incorporating bending, shear and rotary inertia.

The numerical methods that have been used in the solution of this beam problem, when compared with data taken from experiments aboard ships, show in general a good agreement, except for higher modes of vibration (above the fourth) where the measured natural frequencies tend to be lower than the predicted ones. The principal reasons that have been pointed out to explain such discrepancies are the deficient evaluation of parameters such as stiffness, load distribution, added mass, which lead to inaccurate calculations, and the possibility that, after a certain mode, the ship does not behave as a beam anymore [20]. Other possible reasons are the shear lag effect, not taken into consideration by the beam theory, and

the fact of neglecting the coupling between horizontal, vertical and torsional motions, as well as local vibratory responses [21].

Other beam models have been experimented to study the vibratory response of ship hull, such as the three-dimensional beam-shell-sprung body in which the shafting and propeller is modeled as a sprung body connected to a beam-shell representation of the hull. Other examples are those of Kline and Clough [22] who used two beams interconnected by rigid links and springs and the one of Ohtaka [23], who used two elastically connected parallel beams.

The finite element approach proposed by some authors has the advantage of representing the hull by a three-dimensional structure, therefore providing a model closer to reality, without the inconveniences of the one dimensional method, and automatically takes into account the effects of shear lag, local vibratory motions and coupling with torsional vibrations. Unfortunately the finite element method requires a much more elaborate computation technique, because of the need of three-dimensional stiffness factors, wave loads and added mass; besides, the improvement in accuracy for lower modes of vibration is small, and does not seem to justify the increase in computing time.

From what was said above it is concluded that the beam theory is a satisfactory representation of the vibratory response of the ship's structure, provided that no significant response is expected from the higher modes of vibration.

2.2-MODAL ANALYSIS

The modal analysis for the study of the bending moment has the important advantage of leading to a mathematical formulation not too cumbersome, if certain simplifications are assumed. In the modal analysis the equation variables are expressed as a product of two functions, one depending on time only and the other on space only, and the solution is represented in terms of a series of normal modes. It is required an independent solution for the normal modes, which are determined by solving the eigenvalue problems for the natural frequencies and the mode shapes (eigenfunctions).

The vibratory response of a ship structure considered to be an elastic beam with non-uniform mass and free ends is governed by the following partial differential equations, which have been already presented in [24, 25]:

$$\mu\frac{\partial^2 z}{\partial t^2} + c\,\frac{\partial z}{\partial t} + \frac{\partial V_s}{\partial x} = P(x,t) \qquad (1)$$

where:

- t - time
- V_s - Shear force (function of x,t)
- x - horizontal axis in direction of forward motion of the ship (along length of ship)
- z - vertical elastic deflection
- c - structural damping coefficient per unit length (function of x)
- $P(x,t)$-total force per unit length due to ship-wave interaction

μ - effective mass per unit length, which is the sum of the ship's mass (per unit length) and the added mass (per unit length). Function of x only, based on the ship's still water line (high frequency limit).

$$\frac{\partial M}{\partial x} = V_s + I_r \frac{\partial^2 \gamma}{\partial t^2} \qquad (2)$$

where:

I_r - mass moment of inertia per unit length with respect to an axis normal to the xy plane (function of x)

M - bending moment (function of x,t)

γ - elastic deformation angle

$I_r(\partial^2\gamma/\partial t^2)$ - rotary inertia

$$M = EI\frac{\partial \gamma}{\partial x} \qquad (3)$$

where:

E - modulus of elasticity

I - sectional area moment of inertia

EI - bending flexural rigidity

$$\frac{\partial z}{\partial x} = - \frac{V_s}{KAG} + \gamma \qquad (4)$$

where:

KAG - vertical shear rigidity

K - ratio of the average shear stress to the sheer stress at the neutral axis under vertical load

A - section area

G - shear modulus

This system of partial differential equations has to satisfy the boundary conditions at the free ends:

$$V_s(-L/2, t) = V_s(L/2, t) = 0$$

and (5)

$$M(-L/2, t) = M(L/2, t) = 0$$

Reference [24] discusses different possible procedures for the solution of this system of equations for the problem of bow-flare slamming. It was found that the methods of direct solution consisting in converting the partial differential equations into ordinary differential-difference equations, by breaking the beam into a large number of modal segments, were inadequate because of computational limitations.

This method of handling the parameters and variables of the system of equations as discrete rather than distributed has also the disadvantage of requiring the evaluation of the bending and shear rigidity distributions, for which an exact procedure is not readily available. This, along with the approximate numerical methods of solution of the system of partial differential equations, becomes the source of important inaccuracies.

An alternate procedure is possible if the term that includes the rotary inertia I_r can be neglected. In this case the system of equations can be reduced to a single second order linear differential equation, with constant coefficients

and where the parameters EI and KAG (which are both functions of x) are not included.

Since the rotary inertia has a reduced influence on the predominant mode of ship elastic response, which is the first mode of motion, the beam can be represented by a modal model. Reference [20] reports experiments carried out on destroyers, showing that the effect of the rotary inertia on the natural frequency of the vertical vibration is reduced. In [11] the author states that for prismatic bars the influence of the rotary inertia on the natural frequency is a quarter of the influence of shear, this being very small when compared with bending, and therefore carries out the analysis without including the rotary inertia.

Under the assumption that the rotary inertia can be neglected, it is possible to prove that the dynamic behavior of the beam can be treated in terms of a series of responses in each of its normal modes i. Equation (6) represents the orthogonality property between the effective mass per unit length and the normal modes:

$$\int_{-L/2}^{L/2} \mu(x) \cdot X_i(x) \cdot X_j(x) = 0 \qquad (6)$$

where $X_i(x)$ is the normal mode shape of the i^{th} mode; it represents, in arbitrary dimensionless units, the shape of the relative displacement along the length of the beam for the mode i.

The different variables in the system of differential

equations are represented in product form as:

$$z(x,t) = \sum_{i=1}^{\infty} q_i(t) \cdot X_i(x) \tag{7}$$

$$M(x,t) = \sum_{i=1}^{\infty} q_i(t) \cdot M_i(x) \tag{8}$$

$$V_s(x,t) = \sum_{i=1}^{\infty} q_i(t) \cdot V_{s_i}(x) \tag{9}$$

In the preceding equations $q_i(t)$ is the time-varying beam deflection for the i^{th} mode, in a generalized coordinate system.

Hence, equation (7) represents the summation of the contributions from all the modes, obtained for a particular mode i by multiplying the time-varying beam deflection for the i^{th} mode $q_i(t)$ by the dimensionless normal mode function $X_i(x)$. Similarly equation (8) represents the bending moment $M(x,t)$ as the product of $q_i(t)$ by a weighting function $M_i(x)$ and equation (9) gives the shear force $V_s(x,t)$ as the product of $q_i(t)$ by the weighting function $V_{si}(x)$. The spacial functions $M_i(x)$ and $V_{si}(x)$ will be determined later.

The loading function can be represented by the following series expression:

$$P(x,t) = \sum_{i=1}^{\infty} \frac{\mu(x) \cdot Q_i(t) \cdot X_i(x)}{\int_{-L/2}^{L/2} \mu(x) \cdot X_i^2(x) \cdot dx} \tag{10}$$

where $Q_i(t)$ is the weighted forcing function for the i^{th} mode,

and can be found if we multiply both sides of (10) by $X_i(x)$ and, using the orthogonality property (6), integrate over the ship's length:

$$Q_i(t) = \int_{-L/2}^{L/2} P(x,t) \cdot X_i(x)\, dx \tag{11}$$

Combining the fundamental elastic equation (3) with the relation between bending and shear effects (4), neglecting the rotary inertia I_r, and substituting equations (7) through (10) into the initial equations (1) through (4), the following system of differential equations is obtained:

$$\sum_{i=1}^{\infty} [\mu \ddot{q}_i X_i + c\dot{q}_i X_i + q_i \cdot (V_i)' - \frac{\mu Q_i X_i}{\int_{-L/2}^{L/2} \mu X_i^2 dx}] = 0 \tag{12}$$

$$\sum_{i=1}^{\infty} [q_i V_i - q_i (M_i)'] = 0 \tag{13}$$

and

$$\sum_{i=1}^{\infty} [q_i \cdot (X_i)'' + q_i \cdot (\frac{V_i}{KAG})' - \frac{q_i M_i}{EI}] = 0 \tag{14}$$

where the dot means differentiation with respect to time and the prime means differentiation with respect to space (x).

Equations (12), (13) and (14) can be combined into the following expression:

$$\mu \ddot{q}_i X_i + c\dot{q}_i X_i + q_i \cdot [EI \cdot (X_i)'' + EI \cdot (\frac{V}{KAG})']'' =$$

$$= \frac{\mu Q_i X_i}{\overline{\mu}_i} \tag{15}$$

where:

$$\bar{\mu}_i = \int_{-L/2}^{L/2} \mu X_i^2 \, dx \tag{16}$$

Considering the free motion of the beam, with no forcing function present, equation (15) can be expressed as

$$\frac{\ddot{q}_i + (c/\mu)\dot{q}_i}{q_i} = -\frac{1}{\mu X_i}\left[EI\cdot(X_i)'' + EI\cdot\left(\frac{V_i}{KAG}\right)'\right]'' = -\frac{(M_i)''}{\mu X_i} \tag{17}$$

It will be discussed later that c/μ can be assumed constant; therefore, equation (17) contains a function of t alone on the left side, and the right side is a function of x alone. Using the normal procedure of separation of variables on partial differential equations each side is set equal to a constant. In this case, to assure stable responses, the constant is $-\omega_i^2$, where ω_i is the natural frequency of the i^{th} mode.

Setting the first term in (17) equal to $-\omega_i^2$ gives

$$\ddot{q}_i + (c/\mu)\dot{q}_i + \omega_i^2 q_i = 0 \tag{18}$$

which is a linear second order differential equation with constant coefficients.

Setting the last term in (17) equal to $-\omega_i^2$ leads to

$$(M_i)'' = \mu \, \omega_i^2 \, X_i \tag{19}$$

which after integration over the ship's length gives an expression for the spatial weighting function $M_i(x)$:

$$M_i = \int_{-L/2}^{x} \int_{-L/2}^{x} \mu \, \omega_i^2 \, X_i \, dx \, dx \tag{20}$$

This double integral can also be expressed as the following single integral, which is a more convenient form for M_i:

$$M_i = \omega_i^2 \int_{-L/2}^{x} (x-s) \cdot \mu(s) \cdot X_i(s) \, ds \tag{21}$$

Returning to the forced motion, we can combine equations (17) and (19) into (15) to obtain:

$$\overline{\mu}_i \, \ddot{q}_i + C_i \, \dot{q}_i + K_i \, q_i = Q_i(t) \tag{22}$$

where the generalized mass $\overline{\mu}_i$ is defined by equation (16), the generalized damping C_i is

$$C_i = \int_{-L/2}^{L/2} c \cdot X_i^2 \cdot dx \tag{23}$$

and the generalized spring constant K_i is

$$K_i = \omega_i^2 \cdot \overline{\mu}_i \tag{24}$$

From equation (16) we see that $\overline{\mu}_i$ is constant for a particular mode i, because μ, the effective mass per unit length, is only a function of x.

To define the damping coefficient associated with hull vibration we will use here the approach followed in [26], which considers three factors contributing to damping: a structural damping factor per unit length c_s/μ, function of

the mode frequency only, where minor hydrodynamic effects of water friction and generation of pressure waves are also included; the effect of generation of surface waves; and a correction factor due to forward speed, $-V(dA_{33}/dx)$, where A_{33} is the sectional vertical added mass.

Since the ship's structural natural frequencies are relatively high, the effect of damping due to wave generation can be neglected when compared with the other factors included in c_s/μ. We have therefore the following damping, per unit length:

$$c = c_s - V \frac{dA_{33}}{dx} \tag{25}$$

which when multiplied by the square of the normal mode shape of the i^{th} mode X_i^2, and integrated over the ship's length, leads to

$$C_i = (\frac{c_s}{\mu})\overline{\mu}_i - V \int_{-L/2}^{L/2} \frac{dA_{33}}{dx} \cdot X_i^2(x)\, dx \tag{26}$$

The value that should be used for the generalized spring constant K_i is the one obtained if the high frequency asymptotic limit of the added mass is used in (24) because it is assumed that this value has already been reached at the relatively high ship's structural mode frequencies.

From the above discussion, and since the natural frequency of the i^{th} mode ω_i has a fixed value, we conclude that $\overline{\mu}_i$, C_i and K_i are constants. Equation (22) is therefore a linear second order differential equation with constant coefficients

and a forcing function $Q_i(t)$.

For the sake of completeness the closed form solution of this equation is given in (27) for the initial conditions $q_i(0) = \dot{q}_i(0) = 0$, i.e., assuming that at t=0 the ship is at rest.

$$q_i(t) = \int_o^t \frac{Q_i(\tau)}{\lambda_i \mu_i} [e^{-\frac{c}{2\mu}(t-\tau)}] \cdot \sin[\lambda_i(t-\tau)]d\tau \qquad (27)$$

where

$$\lambda_i = \sqrt{\omega_i^2 - \frac{1}{4}\left(\frac{C_i}{\bar{\mu}_i}\right)^2} \qquad (28)$$

and τ is the dummy integration variable.

For the springing phenomenon, the solution for the vibratory response in the frequency domain of equation (22) is readily obtained if the forcing function $Q_i(t)$ is represented in the following form:

$$Q_i(t) = Q_o \exp(j\ \omega_e\ t) \qquad (29)$$

where j is the imaginary unit and the frequency of encounter ω_e is, for head seas,

$$\omega_e = \frac{2\pi}{\lambda}(V+c) \qquad (30)$$

where λ is the wave length, V is the ship forward velocity and c is the wave celarity ($\sqrt{g\lambda/2\pi}$).

The steady state solution of equation (22) is therefore given in complex form by

$$q_i = \frac{Q_o \exp(j\ \omega_e\ t)}{K_i - \omega_e^2 \bar{\mu}_i + j\ \omega_e C_i} \tag{31}$$

which allows the determination of amplitude and phase as a function of the frequency of encounter ω_e.

Equation (21), which was derived assuming that the main contribution to the bending moment comes from the inertial loads along the hull resulting from the vibratory deflections and including the fluid inertial force associated with the added mass, should now be substituted by the following expression, for the midship bending moment:

$$M_s = -\omega_i^2 \int_{-L/2}^{o} x \cdot \mu(x) \cdot X_i(x) \cdot dx \cdot q_i(t) \tag{32}$$

The above expression has only a significant response on the first mode of natural vibration of the ship structure (for i=1), hence:

$$M_s = \int_{-L/2}^{o} x \cdot \mu(x) \cdot X_i(x) \cdot dx \cdot \ddot{q}_i(t) \tag{33}$$

because the influence of the inertial reactions associated with the ship structural deflection accelerations, at frequencies higher than the first mode, will contribute to an increase in the bending moment response for the first mode. On the higher range, the decay of the wave forces exciting the hull deflection will greatly reduce the springing bending moment response. The response should then be given for the first mode by equation (33), with a rapid asymptotic decay for higher natural modes of vibration.

2.3-LOADING FUNCTION

In order to evaluate the wave induced bending moments it is necessary to find, along the ship length, the distribution of the loads due to local wave forces as well as those due to the rigid body motions of the ship.

It is assumed that the influence of the ship rigid body motions on the springing phenomenon is minor, so that the loading force responsible for the high frequency vibration results only from the interaction between the waves and the ship's hull.

The force per unit length is due to hydrostatic and hydrodynamic effects. The hydrodynamic loading includes the inertia force due to body dynamic motions and the dissipative damping force. The hydrostatic term accounts for the exciting effects due to the oncoming waves.

The local vertical wave force acting on a section of a ship is then given by [10].

$$P(x,t)=\{\rho gB\eta^* + [N_z(x)-V\frac{dA_{33}}{dx}]\dot{\eta}^* + A_{33}\ddot{\eta}^*\}e^{-\frac{2\pi\bar{h}}{\lambda}} \qquad (34)$$

where:

- B - local beam
- $N_z(x)$ - local damping force coefficient
- V - ship's forward speed
- A_{33} - local section vertical added mass
- η^* - effective wave profile
- $\bar{h}$ - mean draft of the section, and can be approximated

by

$$\bar{h} = HC_s \tag{35}$$

where:

H - local section draft

C_s - local section area coefficient.

The derivatives $\dot{\eta}^*$ and $\ddot{\eta}^*$ should be calculated as total derivatives, since the rate of change occurs not only in time but also in space:

$$\dot{\eta}^* = \frac{D\eta^*}{Dt} = (\frac{\partial}{\partial t} - V\frac{\partial}{\partial x})\eta^* \tag{36}$$

and

$$\ddot{\eta}^* = \frac{D\dot{\eta}^*}{Dt} = (\frac{\partial}{\partial t} - V\frac{\partial}{\partial x})\dot{\eta}^* \tag{37}$$

Attention is now focused on the evaluation of the effective wave profile η^*.

Previous evaluations of springing response to wave induced high frequency loading have been made considering the undistorted oncoming wave as the water profile responsible for the excitation. It is known that when a ship crosses a wave, there is a very important distortion of the wave profile, the water surface geometry at the ship side being far from similar to the undisturbed wave profile.

The effect of this distortion is even more important if we notice that the amplitude of the distorted wave is reduced as it progresses towards the stern of the ship, therefore creating at the bow a region where the load is concentrated.

Realizing the importance of this phenomenon, Kumai [17] carried out a series of experiments with a self propelled model tanker in order to determine the distortion of the water line and the profiles and envelopes of the wave at ship side. Using this data a loading function was defined, from which the acceleration response at the bow of the ship was evaluated and with the bow acceleration the stress amidship was calculated [16].

An attempt is made here to include a mathematical evaluation of the shape of the distorted wave at ship side in the general procedure for the calculation of the springing response. The outline follows the analysis used by GRIM [27] in 1957 for the calculation of wave generated forces on a ship hull.

When a ship crosses a wave it is clear that the forces produced by the hull on the water surface will generate a deformation which is similar to the exciting wave, oscillating both in time and in space (along the length of the ship). If the wavelength is relatively large compared with the length of the ship, then there is no qualitatively large difference from the heaving motion in still water. The forces produced by the shiphull on the water surface will therefore generate a system of circular waves with a length which is about the same as the length of the exciting wave, and because the magnitude and the direction of the forces produced by the shiphull are not approximately constant over

the length of the ship (as it happens in the case of the heave motion), but change in the same way as the exciting wave, the amplitude of this exciting wave falls off along the ship, resulting in a strong deformation in amplitude, while the frequency of the oncoming wave is not significantly altered.

This deformation has to be taken into account in the calculation of the forcing function (34), and to evaluate the shape of the distorted wave at the ship side the following wave potential in two dimensions will be used as a starting point (the time function $e^{i\omega t}$ is left out):

$$\phi(y,z) = A_o \int_o^\infty \frac{\exp(-Kz)\cos(Ky)}{K-k+i\mu_1} dK +$$

$$\sum_{n=1}^{\infty} A_n \mathrm{Re}\left\{\frac{1}{(y+iz)^{2n}} - \frac{ik}{(2n-1)(y+iz)^{2n-1}}\right\} \qquad (38)$$

where k is the wave number

$$k = \frac{\omega_e^2}{g} = \frac{2\pi}{\lambda} \qquad (39)$$

and μ_1 is a frictional coefficient [28]

$$\mu_1 = \frac{\mu}{c} = \mu\sqrt{\frac{2\pi}{g\lambda}} \qquad (40)$$

with

$$\mu = \nu\ \rho \qquad (41)$$

and

ν - viscosity (L^2T^{-1})

ρ - specific gravity $(M\ L^{-3})$

c - wave celerity

λ - wave length

g - acceleration of gravity

The frictional coefficient μ_1 is always small [29] and usually neglected [30].

The constants A_n in (38) are determined in such a way that the conditions at the boundary are satisfied, or

$$\frac{\partial \phi}{\partial \vec{\eta}} = 0 \tag{42}$$

on the hull surface, where $\vec{\eta}$ is the unit vector normal to the surface.

In three-dimensions, the equation for the potential will be:

$$\phi(x,y,z) = \frac{\exp(i\ \omega_e t)}{2} \sum_{n=0}^{\infty} \int_{-L/2}^{L/2} A_n(\xi) \cdot \psi_n[(\xi-x),y,z]\,d\xi \tag{43}$$

where:

$$\psi_o = \frac{1}{\sqrt{(\xi-x)^2+y^2+z^2}} - k\frac{\pi}{2} e^{-kz} \{N_o[k\sqrt{(\xi-x)^2+y^2}] +$$

$$+\Omega_o[k\sqrt{(\xi-x)^2+y^2}] + \frac{2}{\pi}\int_o^z \frac{e^{ku}}{\sqrt{(\xi-x)^2+y^2+u^2}}\,du +$$

$$+2i\ I_o\ [k\sqrt{(\xi-x)^2+y^2}]\} \tag{44}$$

for n>o

$$\psi_n = \frac{\partial^{2(n-1)}}{\partial z^{2(n-1)}} \left\{ \frac{3z^2}{[(\xi-x)^2+y^2+z^2]^{5/2}} - \frac{1-kz}{[(\xi-x)^2+y^2+z^2]^{3/2}} \right\} \quad (45)$$

and [31]

I_o - Bessel function of 0 order

N_o - Newman function of 0 order

Ω_o - Lommel function of 0 order

The potential (43), for any function $A_n(\xi)$ (ξ is the dummy coordinate associated with the x direction), satisfies the continuity condition and the free surface condition. The constants $A_n(\xi)$ are calculated from (38) so that the boundary conditions on the shiphull are satisfied, and they must disappear beyond the hull's length ($|\xi|>L/2$).

This method is therefore based upon the determination of a source distribution for the two-dimensional cross-sections, the three-dimensional problem being solved from the combination of the pressure distribtuion obtained in two-dimensions; the results should constitute an improvement over the usual strip method.

The potential of the undisturbed wave must be:

$$\phi_w = \frac{hg}{\omega} \exp[i\omega t + ik(x+iz)] \quad (46)$$

If we take off the time factor:

$$\phi_w = \frac{hg}{\omega} \exp[ik(x+iz)] \quad (47)$$

The amplitude of the undisturbed wave, using as reference the plane $z=0$, will therefore be:

$$\eta = he^{ikx} \tag{48}$$

For similarity, in the initial equation (43) for the three-dimensional potential of the distorted wave due to the ship hull, we will use instead of the function $A_n(\xi)$:

$$A_n(\xi) \rightarrow e^{ik} \cdot A_n(\xi) \tag{49}$$

This can be done because the continuity condition and the conditions on the free surface are not violated. As long as the boundary condition is satisfied on the ship hull, the calculation can still be done.

$$e^{ik\xi} = e^{ikx} [1 + (e^{ik(\xi-x)} - 1)] \tag{50}$$

From equation (43), neglecting the time dependence and taking the factor e^{ikx} outside the integral sign, it results:

$$\phi = \frac{e^{ikx}}{2} \sum_{n=0}^{\infty} \{ \int_L A_n(\xi) \cdot \psi_n \, d\xi +$$

$$+ \int_L A_n(\xi) [e^{ik(\xi-x)} - 1] \psi_n \, d\xi \}, \tag{51}$$

which is an interesting form for our analysis. If only the first integral existed, the result would be the same as the one obtained applying the usual strip method to the heave motions. The second integral causes an additional deformation of the wave and constitutes a correction factor.

A first approach to this required correction can be calculated, and leads to the conclusion that the oncoming wave is more deformed as it progresses from the bow to the stern, resulting in a decrease of the effective wave height.

The wave profile using as reference the plane z is calculated from equation (43) using the following r[elat]ionship:

$$\eta^* = \frac{1}{g} \left[\frac{\partial \phi}{\partial t}\right]_{z=0} \tag{52}$$

The values given by (52) should therefore be used in the calculation of the loading function (34).

The analysis was carried out for a ship going in head seas. The derivation of the shape of the distorted wave at ship side is strictly valid for $\omega \to 0$, but acceptable results are expected when using the same analysis for $\omega \neq 0$.

For oblique waves an ap[p]roximate correction can be done considering instead of equation (30)

$$\omega_e = \frac{2\pi}{\lambda_e} (V + c \cdot \cos \alpha) \tag{53}$$

where λ_e is an effective wave length

$$\lambda_e = \lambda \cdot \sec \alpha \tag{54}$$

and λ is the wave length, V the ship's speed, c the wave celerity and α is the angle between the ship's heading and the wave direction, with $\alpha=0$ for head seas.

2.4-PROCEDURE

Since the mathematical procedures of Sections 2.2 and 2.3 are rather long, it was felt it would be convenient to present in an abridged form the results obtained. The purpose of this section is to provide such a summary, as well as indicate how the different parameters in the equations can be calculated.

The bending moment due to springing, evaluated amidships, is given by equation (32)

$$M_s = -\omega_i^2 \int_{-L/2}^{0} x \cdot \mu(x) \cdot X_i(x) \cdot dx \cdot q_i(t) \tag{32}$$

or equation (33)

$$M_s = \int_{-L/2}^{0} x \cdot \mu(x) \cdot X_1(x) \cdot dx \cdot \ddot{q}_1(t) \tag{33}$$

The effective mass per unit length μ is the sum of the ship's mass per unit length (taken from the ship's weight distribution) with the added mass (high frequency limit). The added mass can be calculated using the Frank close-fit method [32].

The normal mode shapes $X_i(x)$ can be calculated using the Prohl-Myklestad digital method mentioned in [33] or the finite element program, DASH [34].

Equation (31) gives a suitable expression for $q_i(t)$:

$$q_i(t) = \frac{Q_o \exp(j \ \omega_e \ t)}{K_i - \omega_e^2 \overline{\mu}_i + j \ \omega_e \ C_i} \tag{31}$$

with

$$Q_i(t) = Q_o \exp(j \ \omega_e \ t) \tag{29}$$

and

$$Q_i(t) = \int_{-L/2}^{L/2} P(x,t)\cdot X_i(x)\, dx \tag{11}$$

For the frequency of encounter ω_e expressions (30) or (53) can be used. K_i is defined by equation (24):

$$K_i = \omega_i^2\, \overline{\mu}_i \tag{24}$$

where the natural frequencies ω_i can be computed using reference [34] or the Prohl-Myklestad method reported in [33]; the definition of $\overline{\mu}_i$ is:

$$\overline{\mu}_i = \int_{-L/2}^{L/2} \mu(x)\cdot X_i^2(x)\, dx \tag{16}$$

C_i is given by equation (26):

$$C_i = \left(\frac{c_s}{\mu}\right) \overline{\mu}_i - V \int_{-L/2}^{L/2} \frac{dA_{33}}{dx} \cdot X_i^2(x)\cdot dx \tag{26}$$

where V is the ship's speed and A_{33} is the added mass (high frequency limit), which can be calculated using reference [32]. The structural damping coefficient (c_s/μ) can be taken from figure 2 of reference [10], of which the following expression is a close fit:

$$\frac{c_s}{\mu} = \left(\frac{\omega}{50}\right)^{9/5} \tag{55}$$

The loading function P(x,t) is given by equation (34):

$$P(x,t)=\{\rho g B\eta^* + [N_z(x)-V\frac{dA_{33}}{dx}]\dot{\eta}^* + A_{33}\ddot{\eta}^*\}e^{-\frac{2\pi\overline{h}}{\lambda}} \tag{34}$$

Here ρ, g, B, V and λ are known. The local damping force coefficient $N_z(x)$ and the local sectional added mass A_{33} can be evaluated using reference [32]. The mean draft $\bar{h}$ is approximated by

$$\bar{h} = H \; C_s \tag{35}$$

with H the local section draft and C_s the local section area coefficient.

The effective wave profile η^* is calculated from (52):

$$\eta^* = \frac{1}{g}\left[\frac{\partial \phi}{\partial t}\right]_{z=0} \tag{52}$$

and (43):

$$\phi(x,y,z) = \frac{\exp(i\omega_e t)}{2} \sum_{n=0}^{\infty} \int_{-L/2}^{L/2} A_n(\xi)\cdot\psi_n[(\xi - x),y,z]d\xi \tag{43}$$

The constants $A_n(\xi)$ are calculated from equation (38) using the boundary condition (42); ψ_n is defined in equations (44) and (45).

In (34), the derivatives $\dot{\eta}^*$ and $\ddot{\eta}^*$ are calculated using definitions (36) and (37):

$$\dot{\eta}^* = \frac{D\eta^*}{Dt} = \left(\frac{\partial}{\partial t} - V\frac{\partial}{\partial x}\right)\eta^* \tag{36}$$

and

$$\ddot{\eta}^* = \frac{D\dot{\eta}^*}{t} = \left(\frac{\partial}{\partial t} - V\frac{\partial}{\partial x}\right)\dot{\eta}^* \tag{37}$$

3. CONCLUDING REMARKS

3.1-DESIGN CONSIDERATIONS

Taking a closer look at the mathematical model developed above, it is possible to draw some conclusions about the importance of some design parameters on the occurrence and magnitude of springing.

Equation (33), which gives the midships bending moment due to springing, shows that a short hull is better than a long one. This is reflected both on the limits of integration and on the mode shape $X_1(x)$, which tends to be larger (amidships and at both ends of the hull) for a long ship. The same conclusion can be drawn using the same reasoning from equation (31) when (29) and (11) are substituted; note that, for a small $q_i(t)$, a high C_i is desirable, and equation (26) indicates again the advantage of a short hull.

The same conclusion about the ship length is obtained if the hull natural frequency of vibration is considered.

The empirical equation (56) is a well-known form for the two-mode natural frequency of ships' hulls [35]:

$$N = \phi \sqrt{\frac{I}{\Delta L^3}} \tag{56}$$

where:

N	- frequency per minute
I	- moment of inertia, in $in^2 ft^2$
Δ	- ship displacement in tons
L	- ship length in feet
ϕ	- empirical coefficient

Therefore the natural frequency is increased by increasing I (by increasing the ship's depth or using mild steel), or by decreasing the product ΔL^3 at a higher rate than I. Since at higher frequencies the wave energy spectrum has smaller values, the excitation will be smaller. This explains the higher values of springing on ships with high block coefficients and long hulls; an important increase in springing is expected in ships longer than 900 ft LBP [10].

Considering $\mu(x)$, which is the sum of the ship's mass with the added mass (per unit length), in equation (33) it is concluded that a small $\mu(x)$ is desirable specially at the points of the hull where $X_1(x)$ is larger. Therefore a fairly uniform weight distribution leads to situations more susceptible to higher springing than a ship with the weight amidships (if the weight and added mass could be concentrated at the nodes of vibration, then there would be no springing bending moment amidships.)

It is interesting to note here that calculations carried out assuming a uniform weight distribution along a ship length lead to a natural frequency 24% lower than the one found using the correct weight distribution [36]. Since lower natural frequencies give higher springing moments, these calculations reinforce the conclusion mentioned on the last paragraph.

It was said before, and noticed on equations (33) and (26) that a small added mass A_{33} is desirable. Experiences carried out with beams of rectangular cross sections show that the

added mass increases as the ratio beam/depth increases. For low values of springing, it is therefore concluded that a small beam/depth ratio is an advantage.

Summarizing the considerations discussed above, the following general recommendations can be made in order to obtain a design that will experience smaller springing stresses:

--short hulls

--small block coefficient

--weight distribution concentrated in the central part of the hull

--small beam/depth ratio

The relative importance of the different parameters mentioned cannot be assessed at this point, and should be investigated by computing the springing response for various design cases and using model tests as well as full scale experiments.

3.2-METHOD EVALUATION

It is important to note that the springing phenomenon results from waves which are short relative to the ship length, while the theorectical considerations to evaluate the wave generated forces on a ship were derived for conditions of long wave lengths compared to the dimensions of the ship cross-section. Therefore the question of the validity of such calculations remain until some further research is done on this matter, because the analytical computations are

dependent upon the accuracy of theorectical expressions that have not been completely verified for the range of high frequency at which springing occurs.

The mathematical model that is the subject of the present study should be evaluated by applying it to different design cases for which experimental data is available. It is suggested that a computer program be written in order to allow the calculation of the springing response of different vessels to different excitations. As an example, it is recommended that the method be applied to the cases reported in [37], where very important experimental data on springing is collected and discussed.

Future investigations on springing response should use some direct measure of the exciting waves, such as a wave probe mounted over the bow of the ship. These measurements would then provide a set of wave records that would be directly correlated with the measured stresses, and would lead to a better understanding of the exciting mechanism of the springing phenomenon as well as allow a positive evaluation of the mathematical models proposed to calculate the springing response.

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